Journal of Basic and Applied Engineering Research
Print ISSN: 2350-0077; Online ISSN: 2350-0255; Volume 2, Number 7; April-June, 2015 pp. 550-554

http://www.krishisanskriti.org/jbaer.html

Mathematical Modeling and Simulation of Missile Target Engagement – 6 DOF Simulation using PN Guidance

Sri Ramya Sanagavarapu[*], Md Shafeeq Ahmed [¥], G Naresh Kumar [£] and Odelu Ojjela [$]

[*]*Postgraduate Student, Department of Applied Mathematics, Defence Institute of Advanced Technology, Pune*
[¥]*Postgraduate Student, Department of Aerospace, Defence Institute of Advanced Technology, Pune*
[£]*Scientist, Defence Research and Development Laboratory, DRDO, Hyderabad*
[$]*Department of Applied Mathematics, Defence Institute of Advanced Technology, Pune*
E-mail: [*]sairamya110@gmail.com

Abstract—*Objective of the study is to mathematically model and simulate the missile-target engagement scenario. Rigid body with 6 Degrees Of Freedom (DOF) is considered for missile dynamics. Target kinematics is simulated as 3DOF with point mass system. Missile-Target engagement is simulated using Proportional Navigation (PN) guidance law. A study has been carried out to investigate the missile dynamic characteristics with different target maneuvers for missile-target engagement. Results show the expected trends. Guidance generated commanded accelerations and LOS rates are within the capability of the missile.*

Keywords: *Modeling and Simulation, Guidance, Proportional Navigation, Missile-Target Engagement, LOS, ZEM*

NOMENCLATURE

u,v,w	Linear velocities in body frame	(m/s)
p,q,r	Angular velocities in body frame	(deg/s)
ψ, ϕ, θ	Euler Angles	(deg)
q_0, q_1, q_2, q_3	Quaternions	-
x,y,z	Positions in inertial frame	(m)
a_c	Commanded acceleration	(m/s^2)
N	Navigation constant	-
λ	LOS angle	(deg)
$\dot{\lambda}$	LOS angle rate	(deg/s)
R	LOS range	(m)
$\dot{R}$	LOS rate	(m/s)
V_t	Target velocity	(m/s)
V_m	Missile velocity	(m/s)
V_c	Closing velocity	(m/s)
t_{go}	Time to go	(s)
ZEM	Zero Effort Miss	(m)
LOS	Line Of Sight	
DOF	Degrees Of Freedom	
PN	Proportional Navigation	

1. INTRODUCTION

Computer-interpretable representations of system structure and behavior are at the center of designing today's complex systems. Mathematical modeling and simulation of dynamical systems plays a major role in design, analysis, verification and validation of systems that may also include hardware in loop [1].

Nonlinear dynamics of the missile is due to its coupled interaction of different fields like flight dynamics and aerodynamics [2]. Complexity of the missile kinematics further increases due to the different phases of missile viz., booster and sustainer.

In this scenario with the target maneuvering, guiding the missile to the target, conserving kinetic energy of the missile becomes a non-trivial task.

PN has been widely used as a guidance law for tactical applications in the recent decades [3]. It is very simple to implement on-board and is very efficient in a wide variety of geometrical situations.

IGLA missile, a class of shoulder launch (man-portable surface-to-air missile to engage low-flying approaching aircraft) system for anti-aircraft has been studied simulating to its similar capabilities. Quantification of kinematic performance was based on commanded acceleration, LOS

angle & its rate and miss distance for different engagement scenarios.

2. PROBLEM DEFINITION

2.1 Objective

Objective of the study is to mathematically model and simulate the missile-target engagement using PN guidance law.

2.2 Six DOF Dynamics Equations

Fig. 1 shows sign convention.

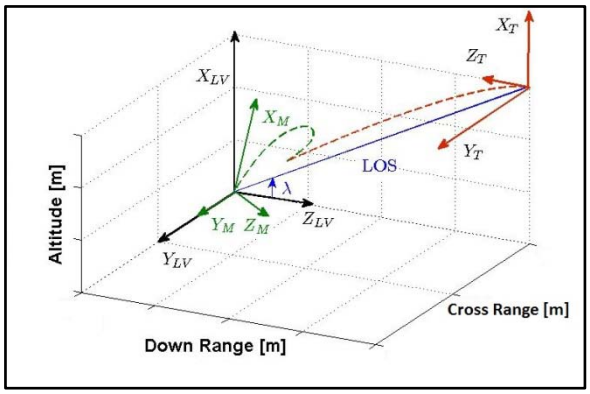

Fig. 1: Sign Convention

Nonlinear, coupled, simultaneous ordinary differential equations given below define the missile six degrees of freedom [4]. These equations are derived from basic Newton laws with respect to the body frame and inertial frame. Quaternions are used for transformation between body frame and inertial frame. Guidance equations are used in realizing missile target engagement scenario [5].

$$\frac{du}{dt} = \frac{F_x}{m} - wq + vr$$
$$\frac{dv}{dt} = \frac{F_y}{m} - ur + pw$$
$$\frac{dw}{dt} = \frac{F_z}{m} - pv + qu$$

Where, F is the net force due to thrust, aerodynamics and gravity contributions.

$$\frac{d\psi}{dt} = \left(q\sin(\psi) + r\cos(\psi)\right)/\cos(\theta)$$
$$\frac{d\phi}{dt} = \left(p + \left(\frac{d\psi}{dt}\right)\right)\sin(\theta)$$
$$\frac{d\theta}{dt} = \left(q\cos(\psi) - r\sin(\psi)\right)$$

$$\frac{dq_0}{dt} = 1/2\left(-pq_0 - qq_2 - rq_3\right)$$
$$\frac{dq_1}{dt} = 1/2\left(-pq_0 - qq_3 - rq_2\right)$$
$$\frac{dq_2}{dt} = 1/2\left(-pq_3 - qq_0 - rq_1\right)$$
$$\frac{dq_3}{dt} = 1/2\left(-pq_2 - qq_1 - rq_0\right)$$

$$\frac{dx}{dt} = q_{11}u + q_{21}v + q_{31}w$$ [1]
$$\frac{dy}{dt} = q_{12}u + q_{22}v + q_{32}w$$
$$\frac{dz}{dt} = q_{13}u + q_{23}v + q_{33}w$$

Guidance equations [6]

$$t_{go} = \frac{R}{\dot{R}}$$
$$R = R_t - R_m$$
$$V_r = V_t - V_m$$

$$ZEM_x = R_x + V_{rx}t_{go}$$
$$ZEM_y = R_y + V_{ry}t_{go}$$
$$ZEM_z = R_z + V_{rz}t_{go}$$

Commanded accelerations

$$a_{cx} = N\dot{\lambda}_x V_c$$
$$a_{cy} = N\dot{\lambda}_y V_c$$
$$a_{cz} = N\dot{\lambda}_z V_c$$

Where

$$\dot{\lambda}_x = \frac{ZEM_{\perp x}}{Rt_{go}}$$
$$\dot{\lambda}_y = \frac{ZEM_{\perp y}}{Rt_{go}}$$
$$\dot{\lambda}_z = \frac{ZEM_{\perp z}}{Rt_{go}}$$
$$V_c = -\dot{R}$$
$$N = 5$$

[1] $q_{11}, q_{12}, q_{13}, q_{21}, q_{22}, q_{23}, q_{31}, q_{32}, q_{33}$ are the elements of directional cosine matrix

2.3 Modeling Assumption

- Missile dynamic equations

1. Rigid body model
2. Symmetric in x-y and x-z plane

- Atmosphere model

3. Density is calculated as exponential decay with altitude

- Gravitational Model

4. Constant acceleration due to gravity (9.81 m/s^2)

3. METHODOLOGY

The primary function of the missile guidance law is to generate steering commands based on strategy which uses the missile and target information as inputs. Guidance laws are classified as classical and modern. Classical guidance laws consist of line-of-sight guidance, pursuit guidance and its variants, proportional navigation and its variants. The modern guidance laws are derived from optimal control theory, differential games, singular perturbation theory, and reachable set theory. Of these, the proportional navigation guidance laws form the boundary between the classical and the modern approach. The basic PN law is a classical guidance law whereas many of its variants are recent extensions and should rightfully be treated as modern guidance laws. PN guidance law is used for the simulations. It dictates that the missile velocity vector should rotate at a rate proportional to the rotation rate of the line of sight in the same direction.

4. MISSILE-TARGET ENGAGEMENT

4.1 Simulation Conditions

- Thrust: Misalignments in the thrust force are neglected.

  Boost phase : 2510 N, t = 0 to 2 sec
  Sustainer phase : 400 N, t > 2 sec

- Aerodynamic terms : Force coefficient in x-direction is 0.3 and zero in other directions.
- Components of acceleration due to gravity is accounted in respective directions.
- Mass is interpolated with time
  Time = [0 ; 2.0 ; 2.1 ; 10] in sec
  Mass = [10.00; 8.00; 7.99; 6.53] in kg

- Missile is assumed with a constant roll rate (p) of 50 deg/sec. Subsequently pitch rate (q) and yaw rate (r) are calculated

$$q = \frac{\left(\frac{F_z}{m} + pv\right)}{u}$$

$$r = \frac{\left(\frac{F_y}{m} + pw\right)}{u}$$

Initial Conditions:
$[u\ v\ w\ p\ q\ r\ \psi\ \phi\ \theta\ q_0\ q_1\ q_2\ q_3\ x\ y\ z] =$

$[30\ ;0\ ;0\ ;\ 50\ ;0\ ;0\ ;\ 0\ ;\ 0\ ;\ -60\ ;\ q_{00}\ ;\ q_{10}\ ;\ q_{20}\ ;\ q_{30}\ ;\ 0;\ 0;\ 0\ ]$

Where

$$q_{00} = \cos\left(\frac{\phi_0}{2}\right)\cos\left(\frac{\theta_0}{2}\right)\sin\left(\frac{\psi_0}{2}\right) - \sin\left(\frac{\phi_0}{2}\right)\sin\left(\frac{\theta_0}{2}\right)\cos\left(\frac{\psi_0}{2}\right)$$

$$q_{10} = \sin\left(\frac{\phi_0}{2}\right)\cos\left(\frac{\theta_0}{2}\right)\cos\left(\frac{\psi_0}{2}\right) - \sin\left(\frac{\phi_0}{2}\right)\cos\left(\frac{\theta_0}{2}\right)\sin\left(\frac{\psi_0}{2}\right)$$

$$q_{20} = \sin\left(\frac{\phi_0}{2}\right)\cos\left(\frac{\theta_0}{2}\right)\cos\left(\frac{\psi_0}{2}\right) - \cos\left(\frac{\phi_0}{2}\right)\sin\left(\frac{\theta_0}{2}\right)\sin\left(\frac{\psi_0}{2}\right)$$

$$q_{30} = \cos\left(\frac{\phi_0}{2}\right)\cos\left(\frac{\theta_0}{2}\right)\cos\left(\frac{\psi_0}{2}\right) - \sin\left(\frac{\phi_0}{2}\right)\sin\left(\frac{\theta_0}{2}\right)\sin\left(\frac{\psi_0}{2}\right)$$

Fourth order Runge-kutta method is used for integrating the above differential equations. Integration time step is 10 milli-seconds. Simulation terminates when LOS rate changes its sign.

4.2 Different Target Maneuvers

Within the feasible engagement domain (Incoming target with a down range of 10km and altitude of 1.5km), different target acceleration profiles with a constant velocity (264 m/sec) are simulated.

1) Zero acceleration (Tacc = 0)
2) One ‘g’ acceleration (Tacc = 1) in + y dir.
3) Two ‘g’ acceleration (Tacc = 2) in + y dir.

Guidance is initiated after the boost phase of the missile.

4.3 Simulation Results

4.3.1 Non-Maneuvering Target

Fig. 1 to Fig. 5 shows various dynamic characteristics of the missile during engagement process for zero target acceleration profile.

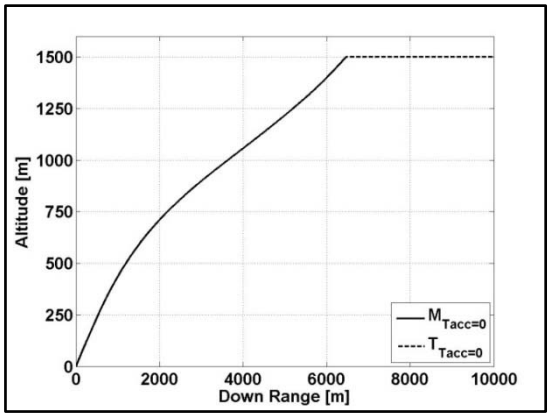

Fig. 1: Variation of Altitude with Down Range

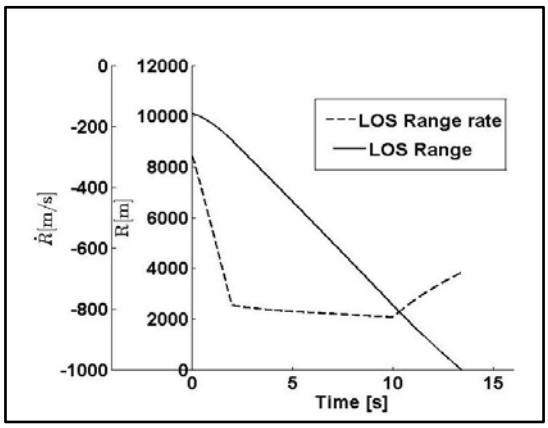

Fig. 2: Variation of LOS Range and LOS Rate with Time

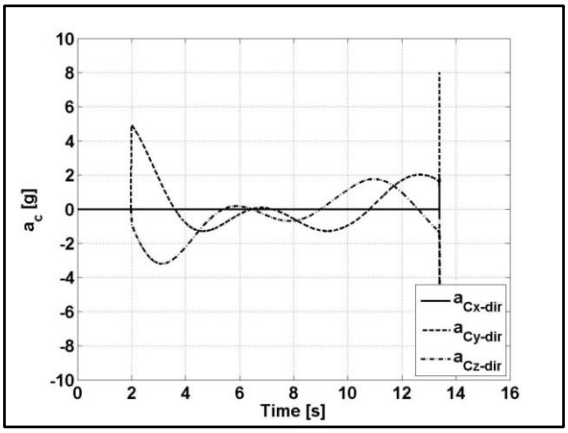

Fig. 3: Guidance Acceleration

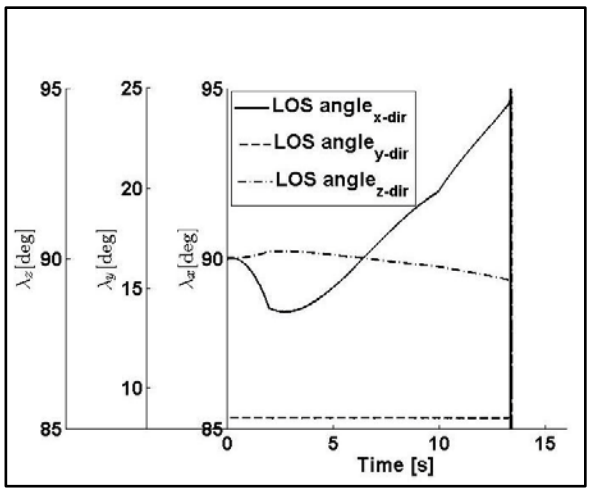

Fig. 4: Variation of LOS angles with Time

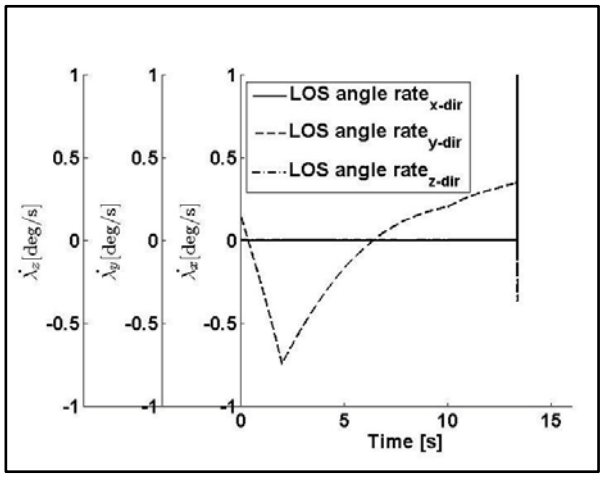

Fig. 5 : Variation of LOS angle rates with Time

4.3.2 Maneuvering Target

Fig. 6 to Fig. 9 shows comparison plots of non-maneuvering target to the maneuvering of target with 1 & 2 g's of lateral acceleration in + y direction.

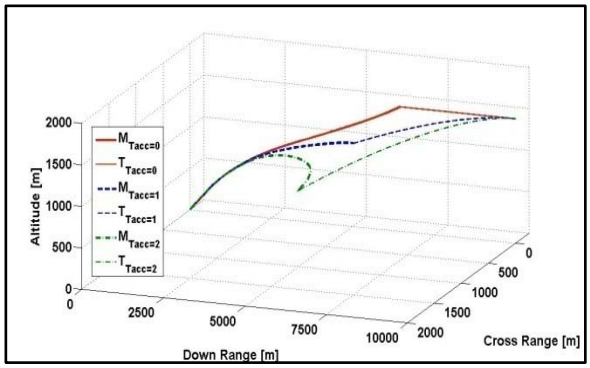

Fig. 6 : Trajectory

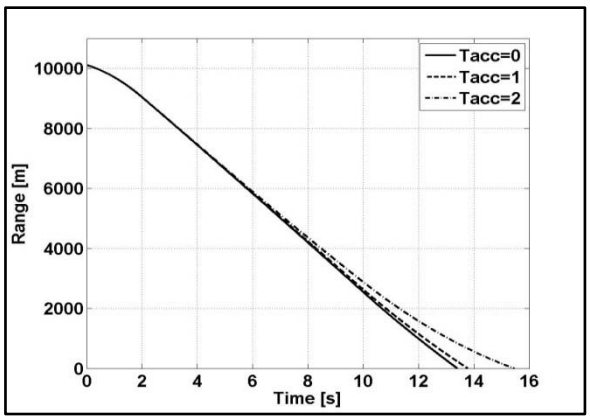

Fig. 7 : Range Variation with Time

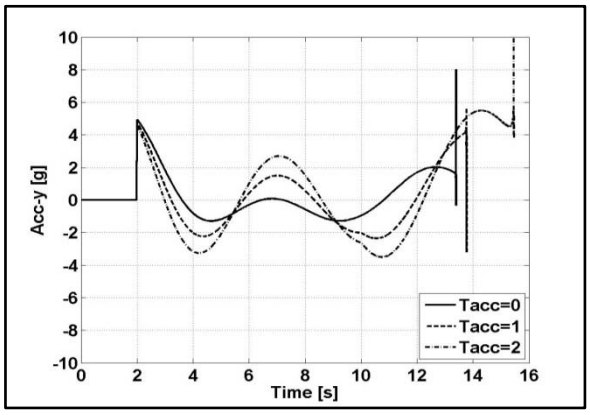

Fig. 8 : Variation of Commanded Acceleration in Y-direction with Time

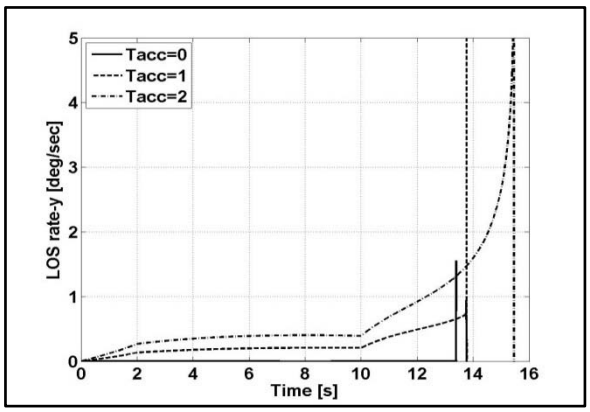

Fig. 9 : variation of LOS angle rate in Y-direction with Time

Observations:

- As the target accelerates in + y direction, missile appropriately guided towards the target as expected
- Time of flight monotonically increases as target acceleration increases
- Maximum commanded acceleration for the simulated case is within 6g
- LOS rate is within 5 deg/sec

Target acceleration (g)	Flight Time (sec)	Miss Distance(m)
0	13.4	1.05
1	13.7	0.59
2	15.4	0.16

5. CONCLUSIONS

Modeling and simulation of the missile-target engagement scenarios for different target maneuvers has been carried out using PN guidance. Missile dynamics is analyzed and found that the commanded accelerations and LOS rates generated are reasonable to achieve in such class of man-portable surface-to-air missiles.

6. ACKNOWLEDGMENT

We express our profound gratitude to Prof.K.K.Mangrulkar, Aerospace Department, DIAT, Pune for encouragement and review of this work. Thanks to all lab mates of the department for their fruitful discussions and help. We would like to acknowledge work done by all the authors of papers referred in this work.

REFERENCES

[1] Bulent Ozkan, “Dynamic Modeling, Guidance, and Control of Homing Missiles”, The Graduate School of Natural and Applied Sciences of Middle East Technical University, September, 2005

[2] Siouris G.M. “Generalized Missile Equations of Motion”, Springer, 2004

[3] Pastrick, H. L., Seltzer, S. M. and Warren, M. E., “Guidance Laws for Short- Range Tactical Missiles”, Journal of Guidance and Control, Vol. 4, No. 2, March- April, 1981

[4] R. M. Gorecki, “A Baseline 6 Degree of Freedom (DOF) Mathematical Model of a Generic Missile”, Weapons Systems Division , Systems Sciences Laboratory, July, 2003

[5] Dipl. -Ing. Karsten Groÿekatthöfer, Dr. -Ing. Zizung Yoon. “Introduction into quaternions for spacecraft attitude Representation”, Technical University of Berlin, 31st May, 2012

[6] Zarchan, P., “Tactical and Strategic Missile Guidance”, New York: AIAA Series, 2007